

Beta Analysis – Theory and Application

What Valuation Professionals Should Know

AGENDA

- Why Beta Matters
- History and Background
- Regression Beta Estimation
- Alternative Estimation Methods
- Unlevering and Relevering Betas
- Debt Betas
- Total Beta
- Conclusions
- Q&A

Why Beta Matters

- Beta, together with the long-term growth rate, often has the largest impact on value in capitalization and DCF methods
- Despite known limitations, beta remains a standard input in cost-of-capital analysis
- In comparison with using an industry risk premium in a build-up method, CAPM and beta can allow for a more tailored approach

A Short History of Beta in Finance

- 1952: Harry Markowitz publishes “Portfolio Selection,” laying the foundation for diversification and portfolio risk
- 1958: James Tobin introduces the risk-free asset as an important component
- 1962: Jack Treynor’s “Market Value, Time and Risk” introduces a Capital Asset Pricing Model
- 1964: William Sharpe’s “Capital Asset Prices” formalizes the CAPM framework
- 1970-1975: Major empirical work by Fischer Black, Myron Scholes, Eugene Fama, Marshall Blume, Michael Jensen, Robert Hamada and others
- 1977: Roll critique highlights that the true market portfolio is unobservable
- 1980s: Focus shifts to estimation problems

Revisiting CAPM

$$R_i = R_f + \beta_i x (R_m - R_f)$$

Where:

R_i = Expected return of security i

R_f = Risk-free rate

β_i = Beta of security i

R_m = Expected return of the total market

Assume $\beta_i = 0$, then the expected return is R_f

Assume $\beta_i = 1$, then $R_i = R_m$

- Beta is the only risk measure that matters under CAPM, because diversifiable risk is assumed away in the theory
- Note that CAPM assumes perfect capital markets, which implies no taxes, no transaction costs, no default costs, and uniform investor expectations
- The CAPM beta is only one interpretation; multi-factor return models use multiple betas

Beta is Risk When Holding Market Portfolio

- The measure of the risk of a security in a large portfolio is defined as the beta (β) of the security
- Beta is formally defined as the covariance of the security with the total market, divided by the variance of the total market

$$\beta_i = \frac{\text{Cov}(R_i, R_m)}{\sigma^2(R_m)}$$

- Beta measures the sensitivity of a security to movements in the market portfolio (i.e., the systematic risk)
- In contrast, volatility is normally defined as the standard deviation of a stock over a period, and is a measure of its total risk

Regression Analysis Background

- Estimates the relationship between an outcome variable (e.g. stock) and one or more explanatory variables (e.g. the market)
- Fits the “best” line (equation) to the observed data by minimizing the sum of squared residuals (difference between actual and predicted values) to calculate a slope coefficient
- Key considerations:
 - Helpful for quantifying relationships
 - Identifies association, not necessarily causation
 - Outliers can materially affect coefficients
 - A high R^2 value does not necessarily mean that the model is meaningful

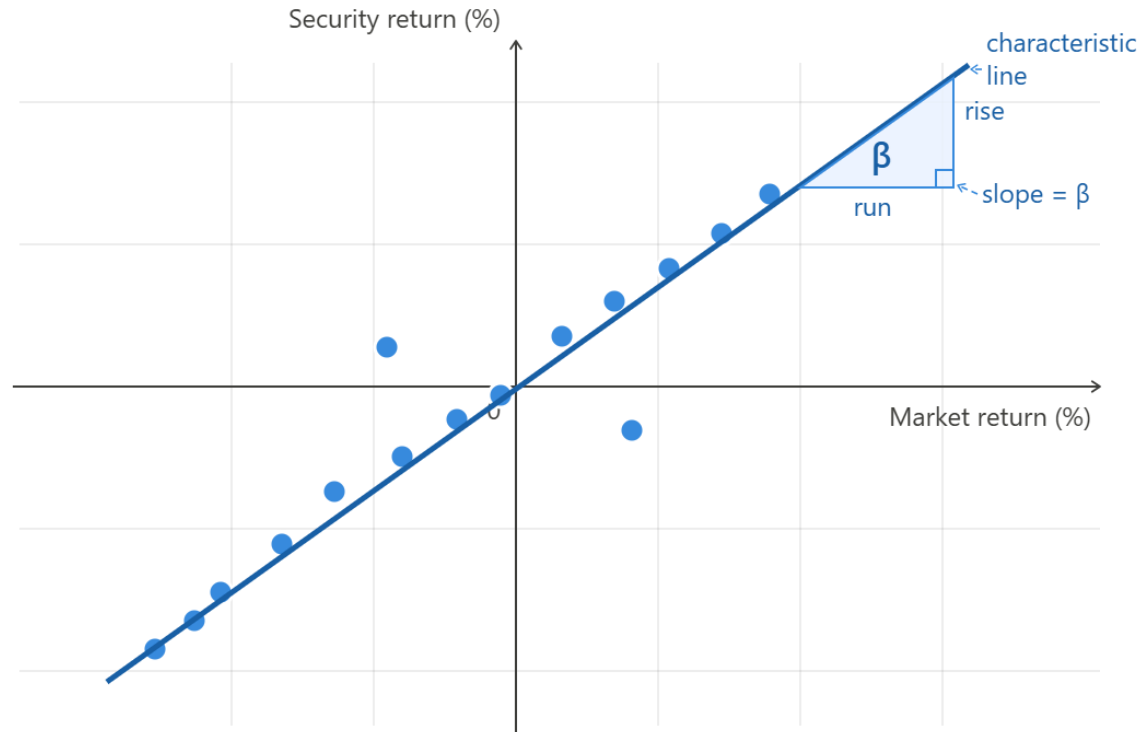
Key Regression Statistics

R^2	Proportion of the variation in outcome variable explained by model (0 to 1)
Standard Error of Regression	Typical size of residual values, a measure of overall fit
t-statistic	Slope coefficient divided by its standard error
p-value	Probability of seeing a result this extreme if the true coefficient were zero
Confidence Interval	Range of plausible values for a coefficient

Estimating Beta with Regression

- Regression of stock returns (R_i) against market returns (R_m)
 - $R_i = \alpha_i + \beta R_m + \varepsilon_i$
 - α = intercept from the regression
 - β = slope of the regression (also Covariance = $(R_i, R_m) / \sigma_m^2$)
 - ε = residual, unexplained portion of returns
- α is the average return not explained by the market over the sample period
- Total stock returns are measured from periodic price changes calculated as logarithmic change, although arithmetic is sometimes used

Estimating Beta with Regression



$$R_i = \alpha_i + \beta_i \cdot R_m + \varepsilon_i$$

Estimating Stock Betas with Regression

- To perform a regression analysis, we need to determine the following:
 - Which index best represents the “market” for the stock?
 - Which time frame to use?
 - Which calculation interval to use for stock prices?
 - Use of logarithmic vs. arithmetic returns
 - Use of total returns vs. returns adjusted for risk-free returns

Beta Regression Analysis Choices

- **Market Definition:**
 - An index that represents the largest portion of market capitalization (S&P 500) is the normal choice and is highly correlated with the total market
 - For small-cap or micro-cap stocks another index may be a better choice (Russell 2000)
- **Time Frame:**
 - Academic research has suggested that a 4 to 6 years time frame is best for estimating a stable long-run beta (but not for a forward-looking beta) – longer windows may improve stability but reduce current relevance
- **Calculation Interval:**
 - For shares that are infrequently traded, using daily (or even weekly) calculations can significantly bias results
 - Monthly calculations are the most commonly used, weekly returns can make sense for liquid stocks, while daily returns can be distorted by thin trading

Beta Estimation in Practice

- Download stock and market index price data (or total return data)
- Choose index, interval, and lookback period
- Estimate beta using regression tools in Excel, Python, Bloomberg, etc.
- Evaluate the output: beta, standard error, R^2 , and economic reasonableness
- Process can easily be automated with Excel or Python

Beta Estimation Problems

- Regression betas are unstable over time
- Betas can be noisy, especially for illiquid, thinly traded, or recently transformed companies
- Some raw betas are not economically credible without further analysis:
 - Very high (>3.0)
 - Negative
 - Close to zero

Regression Beta Adjustments

- There is a reversion to the mean effect for betas, especially for very high and low betas, because they reflect a firm's past business mix and leverage
- Blume adjustment for beta lowers the high values of betas and raises the low values, bringing them all closer to 1.0 with less dispersion, using fixed factors:
 - $\beta_{i2} = 0.343 + 0.667 \times \beta_{i1}$, where β_{i2} is the forecast beta and β_{i1} is the observed beta (last period)
 - Used by some market data providers such as Bloomberg
- Vasicek model adjusts betas toward 1.0 by weighting the historical beta based on its standard error:
 - $$\beta_V = \frac{(\beta_i \times \sigma^2 \beta_0 + \beta_0 \times \sigma^2 \beta_i)}{\sigma^2 \beta_0 + \sigma^2 \beta_i}$$
 - Where β_i is the regression beta and σ is the regression standard error, β_0 (the target prior) is normally assumed to be 1.0 (Bayesian logic)
 - Studies have indicated that the Vasicek betas have slightly better prediction results than unadjusted regression betas
- A beta adjustment will not be appropriate, however, when company has experienced a major structural change, such as acquisitions or distress

Alternative Beta Estimation Methods

- Industry Betas
 - Using an average beta for an industry or group of stocks can address the “noise” in individual stock beta measurements
- Bottom-up betas
 - For a company with multiple lines of business and/or multiple geographies, deriving a beta for each business line or geography and then calculating a weighted average can provide a better estimate
- Fundamental (predictive) betas
 - Derive a beta from current and projected fundamentals of a company, e.g., based on operating or accounting data

Industry Betas

- Aswath Damodaran annually calculates and publishes unlevered betas for ~90 industry sectors (although some have less than 5 companies)
 - For the beta calculations, Damodaran uses a weighted average between 2-year (2/3 weight) and 5-year (1/3) weight betas (based on weekly periods)
- For a narrower or more specific selection, select your own 5+ companies, run regression analyses, unlever the resulting betas, and then average the results

Bottom-Up Betas

- Because beta comes from business risks, it can be difficult to find good direct public comparables for a private company with multiple lines of business and/or geographic markets
- Instead, we can estimate a beta based on the constituent segments of its business, using different groups of public comparables
- When selecting peer companies, operating leverage should be considered in addition to their business mix

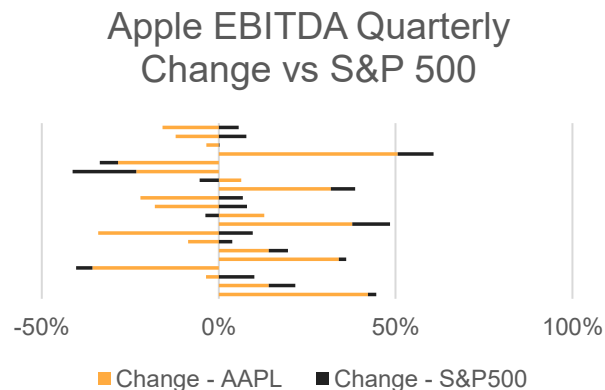
Bottom-Up Beta Illustration:

- Assume a privately held target company with two business segments: industrial distribution (60%), and field service/maintenance (40%)
- If the unlevered betas for the two peer groups are, respectively, 0.8 and 1.2, we can calculate the bottom-up beta of 0.96 as a weighted average
- If the company's target D/E ratio is 40% and tax rate 25%, we can then calculate a bottom-up equity beta of 1.16 (using the Fernandez formula and assuming a debt beta of 0.3)

Fundamental Betas

- Instead of using stock prices, we can use company accounting data to measure performance, e.g. EBITDA
- In some situations, this may be a better measurement of business risk than stock price changes, and can be applied directly to private companies
- However, we may be limited to, at best, quarterly data which may not provide enough measurement points for a regression analysis

- A simple illustration is a regression analysis of Apple's EBITDA over 40 quarters against the S&P 500, which indicates a beta of 1.13, vs. a normal OLS stock beta of 1.09



Betas and Leverage

- Equity betas are impacted by financial leverage, and therefore companies with similar underlying operating risks can have very different equity betas
- In order to isolate the operating (asset) risk we need to adjust for financial leverage, or unlever the equity beta
- Unlevering isolates operating business risk from financial risk, i.e. leverage (but adjustments for operating leverage are more difficult)
- The unlevered (or asset) beta can then be used as a peer comparable for a debt-free company, and relevered for a levered capital structure if relevant

Unlevering and Relevering - Hamada

- The most commonly used formula is Hamada (1972):
 - $B_U = B_L / [1 + (1-T_c) \times (D/E)]$ (with D/E at market value), and $B_L = B_U \times [1 + (1-T_c) \times (D/E)]$
 - Hamada assumes that a) debt is constant in dollar terms, and b) debt beta = 0 (i.e., debt has no default risk)
 - These assumptions may not be appropriate for a growing firm, since the formula implies that D/E declines over time, and/or firms with high leverage
 - The Hamada formula also implies that the tax shield value is proportionate to the debt and is discounted at the cost of debt

Unlevering and Relevering – Harris-Pringle

- Harris-Pringle (1985)
 - $B_U = (B_L + B_D \times D/E) / (1 + D/E)$, and $B_L = B_U + D/E \times (B_U - B_D)$
 - Harris-Pringle is more consistent than Hamada with a constant leverage policy, i.e. for D/EV
 - Also, the tax shield is assumed to have the same risk as the assets and therefore no tax adjustment is included
- “Practitioners Formula” (Harris-Pringle with $B_D=0$)
 - $B_U = B_L / (1 + D/E)$, and $B_L = B_U \times (1 + D/E)$
 - A shortcut method not consistent with finance theory
 - If the actual debt beta is >0 , this formula will provide a higher B_L and therefore a higher WACC

Unlevering and Relevering – Miles-Ezzell & Fernandez

- Miles-Ezzell (1980)

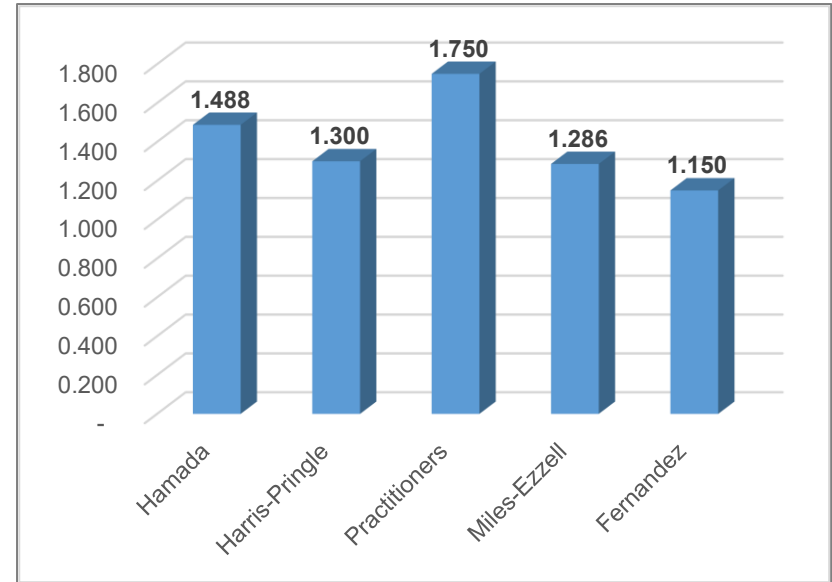
- $B_U = [B_L + (D/E) \times B_D \times (1 - (T_c \times K_d) / (1 + K_d))] / [1 + (D/E) \times (1 - (T_c \times K_d) / (1 + K_d))]$
- $B_L = B_U + (D/E) \times (B_U - B_D) \times (1 - (T_c \times K_d) / (1 + K_d))$
- Miles-Ezzell assumes leverage is rebalanced over time to maintain a D/EV ratio
- Also assumes that the risk of the tax shield is equal to the risk of the firm's assets

- Fernandez (2003)

- $B_U = [B_L + (D/E) \times (1 - T_c) \times B_D] / [1 + (D/E) \times (1 - T_c)]$
- $B_L = B_U + (D/E) \times (1 - T_c) \times (B_U - B_D)$
- Fernandez assumes that debt is constant in dollar terms and that the risk of the tax shield is the same as the debt
- The Hamada formula can be viewed as a special case of Fernandez, where $B_D = 0$

Beta Formula Comparisons

- For firms with a material leverage and a lower credit rating, formulas that do not consider debt betas may produce misleading results
- Fernandez will normally be an appropriate choice for firms expected to have a fixed amount of debt
- Miles-Ezzell or Harris-Pringle will normally be an appropriate choice for firms seeking to maintain a certain leverage ratio, increasing their amount of debt as it grows, with small differences between the two formulas



Levered betas calculated with unlevered beta of 0.7, tax rate 25%, 60/40 D/E ratio, and debt beta of 0.3

Debt Betas

- We can apply the concept of beta to other assets than stocks, including debt
- The beta for a company's debt is often assumed to be zero because of the difficulty in estimating debt betas, but this is a big simplification
- Debt betas can be directly estimated if a company has debt that is actively traded, by regressing historical returns against a market index, just as for stocks, although trading data is limited
- Alternative methods include:
 - Using comparable companies with the same credit rating and publicly traded debt for a regression analysis
 - Approximation based on credit spreads and synthetic credit ratings
 - Using the Merton structural model, which treats equity as a call option on firm assets

Debt Beta Calculation Through Credit Spread

- Heuristic proxy: $\beta_D \approx \text{credit spread} / \text{equity market risk premium}$
- Determine current credit spread (bond yield minus risk-free rate) for relevant bond rating
 - $\beta_D = \text{CS} / \text{ERP} = (r_d - r_f) / (r_m - r_f)$
 - Where CS = credit spread, ERP = equity market risk premium
- For example, assuming a BBB credit rating, we can use the Bank of America BBB US Corporate Index credit spread of 1.01% on 4/27/26. Assuming an equity market premium of 5.0%, we can calculate:
 - $\beta_D = 1.01\% / 5.0\% = 0.24$
- However, we need to consider that the credit spreads reflect compensation not only for default risk, but also liquidity and term risk – therefore, this calculation will typically overstate the debt beta

Debt Beta Calculation Through Credit Spread

- Aswath Damodaran suggested deriving debt betas from historic credit spreads and coverage ratios, but a dedicated debt beta table has not been provided in recent publications
- However, using his current coverage ratios and default spreads, we can estimate debt betas as follows (keeping in mind that they may be overstated):

Rating (Moody's / S&P)	Interest Coverage Ratio >	Coverage Ratio (up to)	Default Spread	Implied Debt Beta
Aaa / AAA	8.50	n/a	0.40%	0.08
Aa2 / AA	6.50	8.50	0.55%	0.11
A1 / A+	5.50	6.50	0.70%	0.14
A2 / A	4.25	5.50	0.78%	0.16
A3 / A-	3.00	4.25	0.89%	0.18
Baa2 / BBB	2.50	3.00	1.11%	0.22

Note: Implied debt beta = CS / 5% ERP, credit data from January 2026

Applying Debt Betas in Practice

- A debt beta of zero is only reasonable for very strong credits (AA/AAA) with modest leverage
- For weaker credits and high leverage cases, a positive debt beta will be a reasonable assumption
- When debt beta is uncertain, an iterative process may be the best approach:
 - Estimate a first-pass debt beta, unlever with that debt beta, re-derive the asset beta, re-compute the debt beta, and repeat until convergence
 - In practice, for investment-grade companies, the error is small because debt beta is small, but for leveraged or distressed situations, the iteration matters

Total Beta

- Background:
 - CAPM beta measures only market risk relevant to a diversified investor
 - For most private company owners, that assumption may be unrealistic because much of their wealth is tied up in one business
 - Total beta is an effort to develop a metric that reflects both market risk and company-specific risk
- Application:
 - Develop a bottom-up company beta from comparable public companies
 - Convert company beta to total beta using the share of total risk explained by the market:
Total Beta = company beta / stock correlation with the market \approx company beta / $\sqrt{R^2}$
 - Then relever the beta, if needed, for the subject company's target capital structure
- Usefulness:
 - As a sanity check or proxy for estimating company-specific risk
 - However, the underlying theory for total beta has been significantly criticized

Conclusions

- Raw regression beta is rarely the whole answer
- Industry or comparable group betas can provide more stability
- Bottom-up beta is usually the most defensible approach for multi-line businesses
- Unlevering and relevering betas must consider firm circumstances and capital structures
- Debt beta matters when leverage and credit risk are meaningful
- Choice of unlevering formula should reflect how the company actually manages its capital structure, not default to Hamada formula
- A small adjustment in beta can have a large impact on cost of equity

Questions / Answers

Appendix

Beta Formula Comparisons

Beta Unlevering:

Formula	BL	Tc	Wd	We	Kd	Bd	BU
Hamada	1.0	25%	40%	60%	N/A	N/A	0.667
Harris-Pringle	1.0	N/A	40%	60%	N/A	0.2	0.680
Practitioners	1.0	N/A	40%	60%	N/A	N/A	0.600
Miles-Ezzell	1.0	25%	40%	60%	7.0%	0.2	0.733
Fernandez	1.0	25%	40%	60%	N/A	0.1	0.700

Beta Relevering:

Formula	BU	Tc	Wd	We	Kd	Bd	BL
Hamada	0.7	25%	60%	40%	N/A	N/A	1.488
Harris-Pringle	0.7	N/A	60%	40%	N/A	0.3	1.300
Practitioners	0.7	N/A	60%	40%	N/A	N/A	1.750
Miles-Ezzell	0.7	25%	60%	40%	10.0%	0.3	1.286
Fernandez	0.7	25%	60%	40%	N/A	0.3	1.150

End-to-End Beta Calculation Example

Scenario: Private industrial distribution company, two-segment business, moderate leverage

Step 1 - Peer group regressions: 6 public peers; 5-yr monthly vs. S&P 500; raw betas 0.85-1.45, average 1.15

Step 2 - Blume adjustment: Adjusted beta = $0.333 + (0.667 \times 1.15) = 1.10$

Step 3 - Peer group debt beta: Peer avg. BBB-rated; Damodaran (Jan 2026) spread 1.11%; implied $\beta_D = 0.21$

Step 4 - Unlever (Fernandez formula): Peer avg. D/E = 40%, Tc = 25%: $\beta_U = [1.10 + 0.40 \times 0.75 \times 0.21] / [1 + 0.40 \times 0.75] = 0.89$

Step 5 - Subject debt beta: Subject rated BB+ (higher leverage); Damodaran spread 1.38%; implied $\beta_D = 0.28$

Step 6 - Re-lever for subject: Subject D/E = 60%, Tc = 25%: $\beta_L = 0.89 + 0.60 \times 0.75 \times (0.89 - 0.28) = 1.16$

Step 7 - Cost of equity: $Ke = R_f + \beta_L \times ERP = 4.5\% + 1.16 \times 5.0\% = 10.3\%$

Key Inputs and Outputs

Peer Group:

Raw avg. equity beta = 1.15

Blume-adjusted beta = 1.10

Peer avg. D/E = 40%

Peer avg. Tc = 25%

Peer debt beta (BBB rated) = 0.21

=> **Unlevered beta (Fernandez) = 0.89**

Subject Company

Target D/E = 60%

Target Tc = 25%

Debt beta (BB+ rated) = 0.28

=> **Re-levered equity beta = 1.16**

$Ke = 4.5\% + 1.16 \times 5.0\% = 10.3\%$

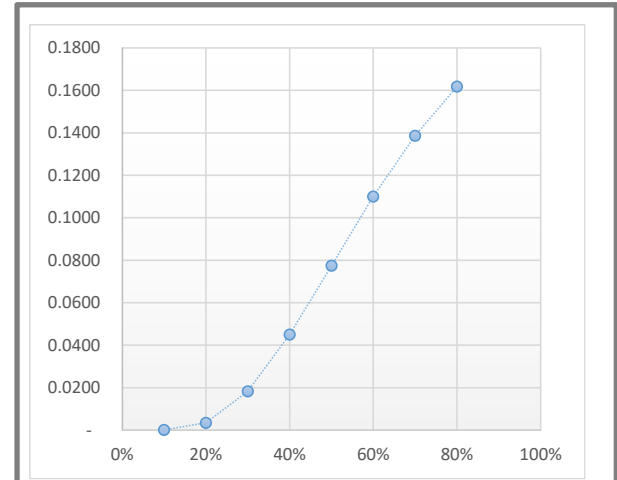
($R_f = 4.5\%$, $ERP = 5.0\%$)

Merton Structural Model (Concept)

- Treats the firm's equity as a call option on its assets, and its debt as a risky bond. Because equity holders have a residual claim, they benefit if asset value rises but are protected by limited liability if it falls. Debt holders effectively hold the assets and have sold a put option to equity holders.
- The systematic risk of debt (its beta) can be derived mathematically from the firm's capital structure and asset risk, without needing traded bond price data
- Debt beta rises as:
 - Leverage increases (more risk accrues to debt)
 - Asset volatility increases (greater chance of default)
 - Bond duration increases (longer exposure to default risk)
 - Credit quality deteriorates (debt behaves more like equity)
- Application: When the company has no publicly traded debt, making regression-based methods unavailable. Particularly useful as a cross-check against credit-spread heuristics.

Debt Beta Estimation With Merton Model

- Merton formula:
 - $\beta_d = [(1 - N(d_1)) / L] \times \beta_a$
 - $d_1 = [-\ln(L) - (y - \sigma_a^2 / 2) \times T] / (\sigma_a \times \sqrt{T})$
- Where:
 - $L = D / (D + E)$
 - $y =$ credit spread
 - $T =$ bond duration (not maturity)
 - $\sigma_a =$ asset volatility
 - $\beta_a =$ asset beta
- Considerations:
 - The model assumes zero-coupon debt, and therefore, duration should be used instead of maturity for the debt
 - Asset volatility and asset beta must be used; using equity volatility and equity beta will overstate debt beta



Debt betas above calculated with Merton Model using $y=1.0\%$, $T=10$, $\sigma_a=18\%$, $\beta_a=0.42$, and varying $D/(D+E)$ – 40% indicates a debt beta of 0.045, roughly consistent with other methods

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